Linearization & Differential Handout

① $f(x) = e^x$, $c = 1$, $f(1) = e$

$f'(x) = e^x$

$f'(1) = e$

$L(x) = e + e(x-1)$

$L(x) = e + e \cdot x - e$

$L(x) = ex$ **B**

② $f(x) = x^2 - 2x + 3$, $c = 2$, $f(2) = 3$

$f'(x) = 2x - 2$

$f'(2) = 2$

$L(x) = 3 + 2(x-2)$

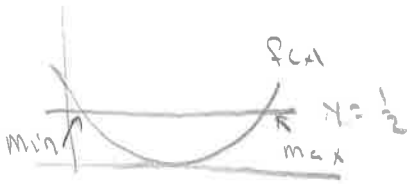
$= 3 + 2x - 4$

$L(x) = 2x - 1$

$|f(x) - L(x)| < .5$

$|x^2 - 2x + 3 - 2x + 1| < .5$

$|x^2 - 4x + 4| < .5$



max $x = 2.707$ **D**

③ $c = 3$, $f(3) = 2$, $f'(3) = 5$

$L(x) = 2 + 5(x-3)$

$L(x) = 5x - 13$

$0 = 5x - 13$

$x = \frac{13}{5} = 2.6$ **C**

④ $y = \tan x$, $x = \pi$

$dy = \sec^2 x \, dx$

$dy = \sec^2(\pi)(.5)$

$= (-1)^2 \cdot .5$

$dy = .5$ **D**

⑤ $f(x) = \sqrt{1-x}$, $a = 0$, $f(0) = 1$

$f'(x) = \frac{-1}{2\sqrt{1-x}}$

$f'(0) = -\frac{1}{2}$

$L(x) = 1 - \frac{1}{2}(x-0)$

$L(x) = 1 - \frac{1}{2}x$

known $x = 1$

For $\sqrt{0.9}$, $x = 0.1$

$L(0.1) = 1 - \frac{1}{2}(0.1)$

$= 0.95$

For $\sqrt{0.99}$, $x = 0.01$

$L(.01) = 1 - \frac{1}{2}(.01)$

$= 0.995$

⑥ a) $y = x^2 + 2x$, $x = 3$, $dx = \frac{1}{2}$

$dy = (2x+2)\,dx$

$dy = 8 \cdot \frac{1}{2}$

$\boxed{dy = 4}$

b) $y = e^{\frac{1}{4}x}$, $x = 0$, $dx = .04$

$dy = \frac{1}{4} \cdot e^{\frac{1}{4}x}\,dx$

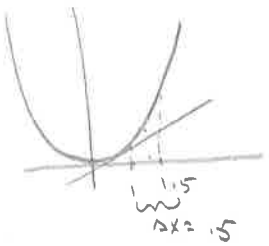
$= \frac{1}{4}(e)^0(.04)$

$\boxed{dy = 0.01}$

⑦ a) $y = x^2$, $x = 1$, $\Delta x = 0.5$

$dy = 2x \cdot dx$

when $x = 1$, $dy = 2(1)(.5) = 1$



$\Delta y = y(1.5) - y(1)$

$\Delta y = \frac{9}{4} - 1 = \frac{5}{4}$

b) $y = \sqrt{x}$, $x = 1$, $\Delta x = 1$

$dy = \frac{1}{2\sqrt{x}}\,dx$

$= \frac{1}{2}(1) = \frac{1}{2}$

$\Delta y = y(2) - y(1)$

$= \sqrt{2} - 1$

⑧ a) $f(x) = x^{2/3}$ $c = 8$ $f(c) = 4$

$f'(x) = \frac{2}{3}x^{-1/3}$

$f'(8) = \frac{2}{3} \cdot \frac{1}{\sqrt[3]{8}} = \frac{1}{3}$

$L(x) = 4 + \frac{1}{3}(x - 8)$

$L(8.06) = 4 + \frac{1}{3}(.06) = \boxed{4.02}$

b) $f(x) = \ln x$ $c = 1$, $f(1) = 0$

$f'(x) = \frac{1}{x}$

$f'(1) = 1$

$L(x) = 0 + 1(x - 1)$

$L(1.07) = \boxed{.07}$

⑨

$A = \pi r^2$, $r = 24$, $dr = .2$

$dA = 2\pi r\,dr$

$dA = 2\pi \cdot 24 \cdot (.2) = \boxed{9.6\pi\ cm^2}$

⑩ a) $\left.\frac{dy}{dx}\right|_{(1,2)} = 3$; $\boxed{y - 2 = 3(x-1)}$

b) $L(x) = 2 + 3(x-1)$

$L(1.1) = 2 + 3(.1) = \boxed{2.3}$

$\frac{d^2y}{dx^2} > 0$ on $1 < x < 1.1$ because $y > 0$ and $x > 0$, so curve is concave up, tangent line below curve, $\Rightarrow$ underestimate

(11) a) $f'(4) \approx \frac{f(5)-f(3)}{5-3} = \frac{-2-4}{2} = \boxed{-3}$

b) $L(x) = -2 + 3(x-5)$

$f(7) \approx L(7) = -2 + 6 = 4$

since $f''(x) < 0$ f is concave down and tangent $L(x) \geq f(x)$ so $f(7) \leq L(7) = 4$

secant line: slope $= \frac{f(8)-f(5)}{8-5} = \frac{3+2}{3} = \frac{5}{3}$

equation: $y - 3 = \frac{5}{3}(x-8)$

$y = 3 + \frac{5}{3}(x-8)$

$f(7) \approx y(7) = 3 - \frac{5}{3} = \frac{4}{3}$

since f is concave down, secant line is below and $f(7) \geq y(7) = \frac{4}{3}$

(12) $r(5) = 30$, $r'(5) = 2$

$L(x) = 30 + 2(x-5)$

$L(5.4) = 30 + 2(.4) = \boxed{30.8 \text{ ft}}$

since r is concave down, tangent line lies above and $L(x)$ is an overestimate